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Matrix elements of the 1.492 MeV beta transition in ^{152}Eu.

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The angular correlation of the $3^- \xrightarrow[1.492\text{MeV}]{\beta} 2^+ \xrightarrow[0.343\text{MeV}]{\gamma} 0^+$ cascade in ^{152}Eu decay was measured as a function of beta energy in the region 950-1400 keV. The present angular correlation results were combined with those on spectrum shape factor and beta-gamma circular polarization available from previous works to determine nuclear matrix elements governing the 1.492 MeV beta transition in ^{152}Eu. The analysis was conducted on a CDC 3600 type computer employing the exact electron radial wavefunctions and taking into account the finite nuclear size effects. A comparison of the matrix elements obtained here with those determined by earlier workers who used the approximate formulae of Kotani, has shown that the present value of $\int \frac{\vec{r}}{\rho}$ is enhanced considerably, while the values of the other vector matrix elements are in agreement with the previous results within experimental errors. Experimentally determined ratio of the $\int i\vec{\alpha}$ to $\int' \vec{r}$ matrix elements is found to be 16.0±5.0, which is not in agreement with the CVC prediction. The energy dependence of the circular polarization of the 343 keV gamma radiation is also predicted at an angle of 157° consistent with the matrix elements reported here.

1. Introduction

Nuclar matrix elements governing the non-unique first forbidden beta transitions have been determined from a knowledge of the various experimental observables using the approximate fomulae due to Kotani, 1959. The formalism of Kotani was based on the point size nucleus assumption and thus higher order terms were neglected so as to facilitate an easy analysis. It has been, however, pointed out (Bühring 1963a, 1963b, 1965) that one canot neglect the higher order terms in the multipole expansion if the usual first forbidden matrix elements are unusually small. In recent years, the availability of Bhalla and Rose tables (Bhalla & Rose 1962) for the parameters of the exact electron radial wavefunctions and the beta decay formalism due to Bühring including the higher order terms, have enabled an accurate analysis of the experimental data for finding the matrix elements. Such an attempt was made to analyze some $3^- \rightarrow 2^+$ beta transitions in Ga^{72} and La^{140} (Newsome & Fischbeck 1964) and La^{140} (Singru *et al* 1966). Simms, (1964,) has arranged the various theoretical expressions for beta decay observables in a convenient from for the analysis of $2^- \rightarrow 2^+$

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beta transitions in ^{84}Rb and ^{86}Rb. Singru *et al* 1966 have adopted the same procedure in the case of ^{140}La. The findings of the various authors appear to justify the importance of finite nuclear size effects and the usage of exact expressions for the evaluation of matrix elements.

The decay scheme of ^{152}Eu is well established and is shown in figure 1. The 1.492 MeV beta transition ($3^- \rightarrow 2^+$) in ^{152}Eu decay has been the subject of study by various authors. The features of this transition are a high log ft value 12.2 (Nathan & Hultberg 1959, Schneider 1960), a large spectrum shape deviation (Langer & Smith 1960) and a large beta-gamma anisotropy (Feschbeck & Wilkinson 1960, Bhattacharjee & Mitra 1960,

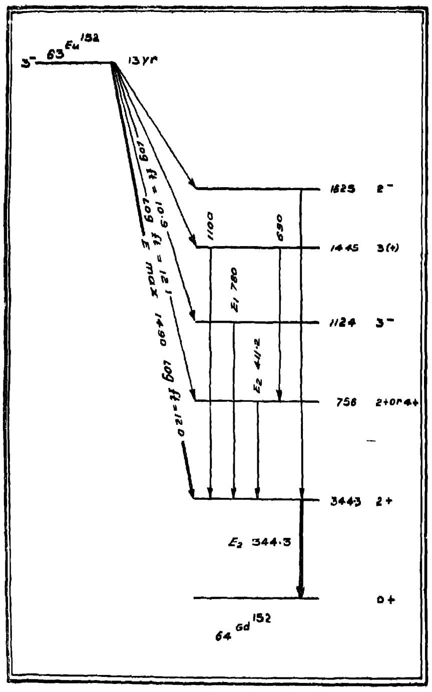

Figure 1 : Decay scheme of ^{152}Eu.

Dulaney *et al* 1960 and Alexander & Steffen 1962). Many authors (Bhattacharjee & Mitra 1960, Alexander 1962, Dulaney *et al* 1964 Lipnik & Sunier 1964) have extracted matrix elements that characterize the 1.492 beta transition of ^{152}Eu from a knowledge of the experimental observables and using the approximate expressions of Kotani. The present work is undertaken to investigate the matrix elements governing the same beta transition in ^{152}Eu, employing the electron radial wave functions of Bhalla and Rose and taking into account the finite nuclear size effects. The procedure adopted for the extraction of matrix elements was due to Simms (1965). We have also, incidentally, measured the energy dependence of angular correlation of 1.492 (β) — 0.343 MeV (γ) cascade in ^{152}Eu (figure 1) with our correlation set-up. The present data on angular correlation were combined with the data on spectrum shapefactor (Langer 1960) and beta-gamma circular polarization correlation (Alexander 1962) for finding the matrix elements. The analysis for the matrix elements was carried out on the CDC 3600 computer at Tata Institute of Fundamental Research, Bombay. The present results are compared with those of earlier workers who followed Kotani's formalism. The energy dependence of the circular polarization of the 0.343 MeV gamma following the beta at 157° is also predicted consistent with the present values of matrix elements, so as to facilitate a comparison as and when the experimental data on this function are available.

2. Experimental

The europium-152 source was obtained as luquid $EuCl_3$ in HCl from the Atomic Energy Establishment, Harwell (U. K.). A drop of liquid was deposited on a mylar film of thikness 0.6 mg/cm^2 for the present experimentation. The angular correlation of the $3^- \xrightarrow[1.492\,\mathrm{MeV}]{\beta} 2^+ \xrightarrow[0.343\,\mathrm{MeV}]{\gamma} 0^+$ cascade was measured with a fast-slow scintillation spectrometer described earlier (Rao *et al* 1965, Rao *et al* 1966) as a function of beta energy in the range 950-1400 keV. All the usual corrections were applied to the observed data and the differential correlation coefficients $\epsilon(\omega)$ in their final form are shown in figure 2 as a function of beta energy. These results are in substantial agreement with those reported by Alexander (1962).

3. Analysis

The experimental data used for the determination of matrix elements are shown in figures 2, 3 and 4. The shape-factor $C(W)$ and the beta-gamma circular polarization $P_\gamma(\theta)$ functions shown in figures 3 and 4 were taken from Langer (1960) and Alexander (1962) respectively. The search for the matrix elements which gave the best fit to all the experimental data, simultaneously, was made on the CDC 3600 computer.

One finds the relevant details concerning the extraction of matrix elements in several earlier papers (Newsome & Fischbeck 1964, Simms 1965) and so these are not given here again. The $3^- \rightarrow 2^+$ beta transition is caused by three matrix elements of rank 1 and one matrix element of rank 2. The matrix element parameters in Kotani's notation are given below :

$$z = C_A \int B_{ij}. \qquad \rightarrow \lambda = 2$$

$$\left.\begin{aligned} y &= -\frac{-C_V \int i\vec{\alpha}}{C_A \int B_{ij}} \\ x &= \frac{-C_V \int \vec{r}}{C_A \int B_{ij}} \\ u &= \frac{C_A \int i\vec{\delta} \times \vec{r}}{C_A \int B_{ij}} \end{aligned}\right\} = 1$$

The rank 1 matrix elements expressed above are relative to z.

$Y = \xi' y - \xi (x + u)$, a linear combination of rank 1 matrix elements. ξ' distinguishes the relativistic matrix elements from the nonrelativistic ones. And $\xi' \approx \xi$. The computer analysis wase made for x, u and Y using the data shown in figures 2, 3 and 4. The standard matrix element can be known from the logft value of the beta transition and hence the absolute values of the rank 1 matrix elements can be calculated.

The parameters of the electron radial wave function that would occur in the expressions for ϵ (W), $C(W)$ and $P_\gamma(\theta)$ were determined using the Bhalla and Rose tables. In the first instance a coarse search was made for x, u and Y to know the approximate range of each parameter that explained all the experimental data, simultaneously. Afterwards a fine search was made in steps of 0.01 to get well defined solutions. The criterion for the acceptance of each solution was decided by the χ^2 test of the experimental data. The computer was instructed to print out only these sets of matrix element parameters yielding values of ϵ (W), $C(W)$ and $P_\gamma(\theta)$ that agreed with the experimental results with a more than 30% probability. Thus finally the following ranges consistent with the

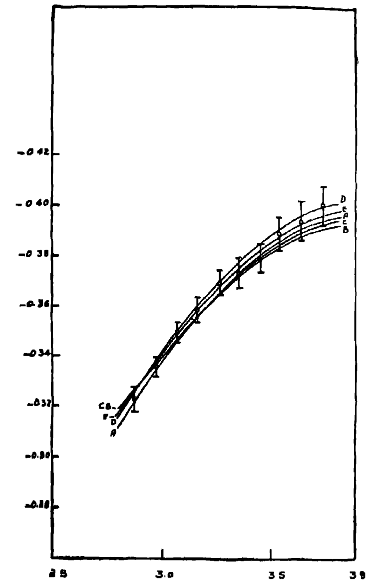

Figure 2 : β-γ angular correlation function ϵ (W) as a function of beta energy (m_0c^2 units). The points with vertical flags represent the experimental values. The curves are the theoretically predicted functions for the sets of matrix element parameters given in table 1.

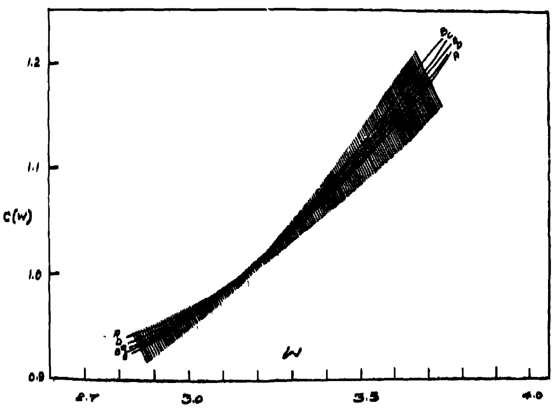

Figure 3 : The experimental shape of $C(W)$ function taken from Langer (1960). The shading indicates the errors while solid lines are the energy dependence generated by the matrix element parameter sets given in table 1.

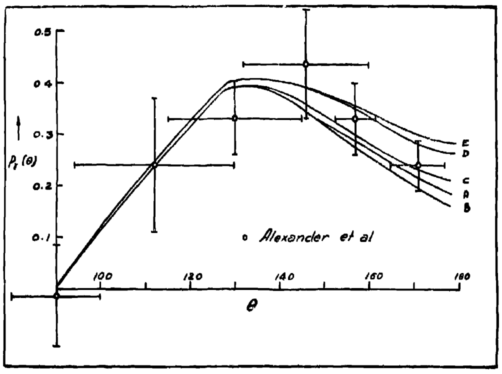

Figure 4 : The experimental data on β-γ circular polarization correlation taken from Alexander (1962), at E_β (average)=3.2 (m_0c^2 units). The solid lines are the theoretical $P_\gamma(\theta)$ generated by the matrix element parameter sets given in table 1.

experimental data shown in figures 2, 3 and 4 were obtained.

$$z = 1.00$$

$$0.47 \leqslant x \leqslant 0.58$$

$$0.08 \leqslant u \leqslant 0.00$$

$$0.60 \leqslant Y \leqslant 0.85.$$

Some sets of matrix element parameters are given in table 1 and the corresponding theoretical functions ϵ (W), $C(W)$ and $P_\gamma(\theta)$ are shown in figures 2, 3 and 4, respectively. From these figures one finds a good agreement between the theoretical and experimental values.

The prediction of the energy dependence of the 1.492 MeV beta - 0.343 MeV gamma circular polarization correlation was also included as a part of the computer programme, for each satisfactory set of parameters. In figure 5, the behaviour of P_γ (W) at θ=157° is shown as a function of beta energy for each of the sets given in table 1. An experiment of this kind may be helpful to obtain more well defined ranges for x, u and Y.

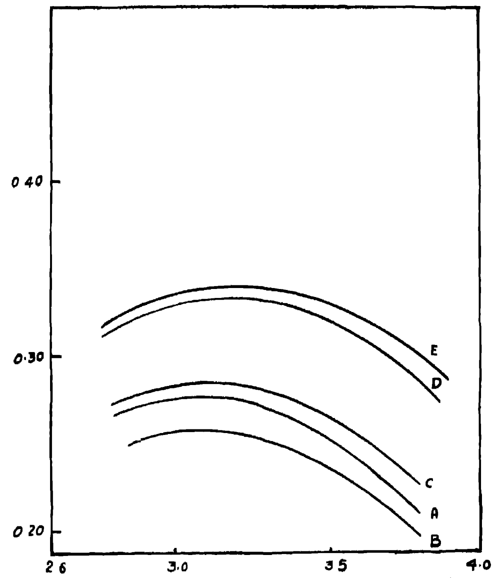

Figure 5 : The theoretically predicted energy dependence of β-γ circular polarization $P_\gamma(W)$ at $\theta = 157°$ shown as a function of beta energy for different sets of matrix element parameters given in table 1.

TABLE 1. SOME SETS OF MATRIX ELEMENT PARAMETERS WHICH GIVE THE BEST FIT TO THE EXPERIMENTAL DATA. $Z = 1.0$

Designation.	Y	x	u
A	0 65	0.55	0.08
B	0.66	0.56	0.04
C	0.70	0.56	0.05
D	0.74	0.53	0.04
E	0.79	0.52	0.01

TABLE 2. ABSOLUTE VALUES OF MATRIX ELEMENTS GOVERNING THE 1.492 MEV BETA TRANTITION OF ^{152}Eu

Reference.	$\frac{\lvert\int B_{ij}\rvert}{\rho}$ $(\times 10^{3})$	$\frac{\lvert\int \vec{r}\rvert}{\rho}$ $(\times 10^{3})$	$\frac{\lvert\int \vec{i\sigma}\times\vec{r}\rvert}{\rho}$ $(\times 10^{3})$	$\lvert\int \vec{i\alpha}\rvert$ $(\times 10^{3})$
Present results.	3.1±0.4	2±0.2	0.13±0.13	0.51±0.17
Alexander, 1962.	2.9±0.4	0.43±0.25	0.36±0.19	0.25±0.11

The theoretical ratio Λ_{CVC} of the vector matrix elements $\vec{i\alpha}$ to $\int\vec{r}$ based on the conserved vector current hypothesis as derived by Fugita (1962), is

$$\Lambda_{CVC} = \frac{1.2\alpha Z}{\rho} + (W_o - 2.5)$$

$$= 2.4\xi + (W_o - 2.5)$$

(α is the fine structure constant, Z is the charge of the daughter nucleus and ρ is the nuclear radius) ; for ^{152}Eu$\xi = 13.53$ and $\rho = 1.7 \times 10^{-2}$ natural units. Thus one gets Λ_{CVC} (theoretical) = 33.0 for the present beta transition.

Λ_{CVC} (experimental) obtained from the values of x, u and Y is 16.0±5. Thus one sees no agreement between the experimental and theoretial values of Λ_{CVC}. The experimental value of Λ that follows from Kotani's expressions was found to be in agreement with the theoretical one within experimental errors as reported by Alexander (1962). However, the error limits of this work are very wide. The experimental value of Λ for a similar transition in ^{140}La is also repored to be not in agreement with the theoretical prediction (Singru *et al* 1966). The inclusion of third forbidden matrix elements in the analysis might bring the experimental *CVC* ratio into agreement with the theoretical one. However, the analysis of beta transitions including the third forbidden matrix elements is rather a difficult task as they introduce a separate energy dependence of the observables. Further, it is doubtful how far the present experimental accuracies warrant the inclusion of third forbidden matrix elements in the beta decay theory.

4. Absolute values of the matrix elements.

The standard matrix element $C_A \int B_{ij}$ can be evaluated using the log ft value of the 1.492 MeV beta transition of ^{152}Eu, from the relation

$$| C_A \int B_{ij} \ |^2 = \frac{\Pi^3 ln2}{f_c t}$$

where $f_c t$ is the corrected value for the non-statistical shape of the beta spectrum. $f_c t = 10^{11.9}$ for the present beta transition. Thus one gets $\int B_{ij}=5.2+0.7\times10^{-5}$ natural units. Finally, one gets the absolute values of the vector matrix elements from a knowledge of x, u, Y and $\int B_{ij}$. In order to have a more significent comparison, the matrix elements containing r were divided by the nuclear radius ρ and are given in table 2 along with those of Alexander & Steffen (1962), whose values, in a way, represent all the earlier works.

5. Discussion.

From table (2) it can be seen that the present value of $\vec{r}/\rho$ is considerably enhanced while $i\vec{\alpha}$ and $i\vec{\sigma}\times\vec{r}/\rho$ values agree within experimental errors with the previous results. However the large uncertainties in the values of matrix elements do not allow a significant comparison. It also shows the masking of higher order effects by the large experimental errors in different observables. For the present case an experiment on the energy dependence of β-γ circular polarization will be helpful to narrow down the limits of matrix elements reported here. For this the theoretical energy dependence of β-γ circular polarization correlation functions given in this work will be helpful for a future experimenter to compare the experimental results with the theoretical predictions.

The values of rank one matrix elements suggest that one cannot apply the modified B_{ij} approximation (Kotani 1959) to the 1.492 MeV beta transition of ^{152} Eu, which requires x and u to vanish. The B_{ij} value characterizing the present beta transition has of course suffered less reduction in size than the rank one matrix elements. The same conclusion follows from the previous values of the matrix elements also. Finally it may be concluded from the present value of $\int \frac{\vec{r}}{\rho}$ that it is important to take into account the finite nuclear size effects in beta decay theory for the determination of matrix elements. However, the full advantage of the inclusion of higher order effects in the theory may not be derived unless the experimental accuracies improve.

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